Factorization in Semi-Inclusive Polarized Deep Inelastic Scattering*

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Abstract

We calculate and analize the $\mathcal{O}(\alpha_s)$ one-particle inclusive cross section in polarized deep inelastic lepton-hadron scattering, using dimensional regularization and the HVBM prescription for γ_5. We discuss the factorization of all the collinear singularities related to the process, particularly those which are absorbed in the redefinition of the spin dependent analogue of the recently introduced fracture functions. This is done in the usual $\overline{MS}$ scheme and in another one, called $\overline{MS_p}$, which factorizes soft contributions and guarantees the axial current (non)conservation properties.

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Introduction

In recent years, there has been an increasing theoretical and experimental interest in semi-inclusive deep inelastic phenomena. Specifically, the use of one-particle inclusive measurements, with polarized targets and beams, has been indicated as an adecuate tool to unveil the spin structure of the proton, elusive to the totally inclusive experiments (see [1, 2, 3] and references therein).

However, the available calculations [4, 5, 6] of one-particle inclusive polarized deep inelastic cross sections do not include the full QCD next to leading order corrections, which are essential to weight the role of the gluon polarization [7]. These calculations are also not adequate for phenomenological purposes because they are not able to describe the full target fragmentation kinematical region [8], that, incidentally, is expected to be favored in the foreseen experiments [1]. Higher order corrections produce singularities in this region that are usually avoided imposing cuts in the transverse momentum allowed for the produced particles [8, 9].

In order to cope with the problem of the target fragmentation, a new factorization approach for semi-inclusive processes has been introduced by Trentadue and Veneziano [10], defining new unpertubative distributions, called fracture functions. These distributions measure the probability for finding a parton and a hadron in the target and can be measured in the proposed experiments. The use of this approach in next to leading order one-particle inclusive unpolarized deep inelastic scattering, has been recently shown [9] to allow a consistent factorization of the collinear singularities coming from the kinematical region where the hadron is produced in the direction of the incoming nucleon, and which cannot be absorbed in the redefinition of the usual distributions.

The extension of this approach to polarized phenomena using dimensional regularization [11], implies an arbitrariness regarding the definition used for the γ_5 matrix. Between the different prescriptions, the one proposed in reference [12] (HVBM), has been proved to be fully consistent and extensible to any order in perturbation theory. However, this prescription introduces finite soft contributions that come from the breaking of chiral invariance and have to be substracted in the distribution functions, withdrawing from the $\overline{MS}$ scheme [13, 14]. It is important, then, to show explicitely that the substraction rule used for polarized parton distributions in totally inclusive processes factorizes the same singularities and soft terms in those which are one-particle inclusive and can be generalised for fracture functions in a completely consistent way.

In the following section we define the spin dependent one-particle inclusive cross section in terms of the polarized structure and fracture functions and the unpolarized fragmentation function. In the third

we show the results for the unsubstracted $\mathcal{O}(\alpha_s)$ contributions coming from the relevant diagrams using the HVBM prescription. Finally, we discuss the factorization of collinear singularities and the rules for the substraction of finite soft terms in the different factorization schemes.

Definitions and kinematics.

In this section we introduce the spin dependent fracture function, generalizing what has been done in references [9, 10], and we establish our notation.

In the one photon exchange approximation for the interaction between a lepton of momentum l and helicity λ and a nucleon N of momentum P and helicity λ', the differential cross section for the production of n partons can be written as:

$$\frac{d\sigma^{\lambda\lambda'}}{dx\,dy\,dPS^{(n)}} = \sum_{i=q,\bar{q},g}\sum_{\lambda''=\pm1}\int\frac{d\xi}{\xi}\,P_{i/N}(\xi,\frac{\lambda''}{\lambda'})\,\frac{\alpha^2}{S_H x}\,\frac{1}{e^2(2\pi)^{2d}}\left[Y_M(-g^{\mu\nu})+Y_L\frac{4x^2}{Q^2}P_\mu P_\nu+\lambda Y_P\,\frac{x}{Q^2}\,i\epsilon^{\mu\nu qP}\right]H_{\mu\nu}(\lambda'') \tag{1}$$

where

$$x=\frac{Q^2}{2P\cdot q},\quad y=\frac{P\cdot q}{P\cdot l},\quad S_H=(P+l)^2 \tag{2}$$

being q the transfered momentum ($Q^2=-q^2$) and $dPS^{(n)}$ the phase space of n final state partons in $d=4-2\epsilon$ dimensions. $P_{i/N}(\xi,\lambda''/\lambda')$ is the probability for finding a parton i with helicity λ'' carrying a fraction ξ of the nucleon momentum, and the kinematical factors appearing in the leptonic tensor are

$$Y_M=\frac{1+(1-y)^2}{2y^2}\qquad Y_L=\frac{4(1-y)+(1-y)^2}{2y^2}\qquad Y_P=\frac{2-y}{y} \tag{3}$$

The helicity dependent partonic tensor is defined by

$$H_{\mu\nu}(\lambda'')=M_\mu(\lambda'')\,M_\nu^\dagger(\lambda'') \tag{4}$$

where M_μ is the parton-photon matrix element, with the photon polarization vector factorized out.

In order to isolate the antisymmetric part of this tensor, which leads to the polarized structure function, we take the difference between cross sections with opposite target helicities

$$\Delta\sigma\equiv\sigma^{\lambda+}-\sigma^{\lambda-} \tag{5}$$

at variance with the unpolarized case where an average over beam and target helicities is taken. With these definitions

$$\frac{d\Delta\sigma}{dx\,dy\,dPS^{(n)}}=\sum_{i=q,\bar{q},g}\int\frac{d\xi}{\xi}\,\Delta P_{i/N}(\xi)\,\frac{\alpha^2}{S_H x}\,\frac{1}{e^2(2\pi)^{2d}}\,\lambda Y_P\,\frac{x}{Q^2}\,i\epsilon^{\mu\nu qP}\Delta H_{\mu\nu} \tag{6}$$

where

$$\Delta H_{\mu\nu} \equiv M_\mu(+)M_\nu^\dagger(+) - M_\mu(-)M_\nu^\dagger(-) \tag{7}$$

$$\Delta P_{i/N}(\xi) \equiv P_{i/N}(\xi,+) - P_{i/N}(\xi,-) \tag{8}$$

In analogy with the unpolarized case, treted in reference [9], we write the cross section for the production of unpolarized hadrons h of energy E_h with polarized beams and targets, differential in the variable $z = E_h/E_N(1-x)$ as

$$\frac{d\Delta\sigma}{dx\,dy\,dz} = \int \frac{du}{u} \sum_N \sum_{j=q,\bar{q},g} \int dPS^{(n)}\, \frac{\alpha^2}{S_H x}\, \frac{1}{e^2(2\pi)^{2d}}\, \lambda Y_P\, \frac{x}{Q^2}\, i\epsilon^{\mu\nu qP}\, \Delta H_{\mu\nu}$$
$$\left\{ \Delta M_{j,h/A}\left(\frac{x}{u}, \frac{E_h}{E_A}\right)(1-x) + \Delta f_{j/A}\left(\frac{x}{u}\right) \sum_{i_\alpha=q,\bar{q},g} D_{h/i_\alpha}\left(\frac{E_h}{E_\alpha}\right) \frac{E_h}{E_\alpha}(1-x) \right\} \tag{9}$$

The variable u is given, as usually, by $u = x/\xi$. The spin dependent fracture function $\Delta M_{j,h/N}(\xi,\zeta)$ is the probability for finding a polarized parton j with momentum fraction ξ and a hadron h with momentum fraction ζ in the nucleon N. Both $\Delta f_{j/N}$ and D_{h/i_α} are the usual spin dependent parton distribution and fragmentation function respectively [15]. Notice that in the case of hadrons with spin, the fragmentation function is exactly the unpolarized one due to the fact that we are summing over the final state polarizations because they are not observed in the present experiment. For spinless hadrons, this is also true provided the fragmentation mechanism is independent of the helicity of the parent parton, as it is usually assumed [4, 5].

$\mathcal{O}(\alpha_s)$ contributions.

In the following we calculate the spin dependent cross section up to order α_s. For this purpose, it is convenient to use the same kinematical variables as in the unpolarized case but contracting the matrix elements $\Delta H_{\mu\nu}$ of the relevant processes with the following projector

$$P^{\mu\nu}_{pol} \equiv \frac{\alpha^2}{S_H x}\, \frac{1}{e^2(2\pi)^{2d}}\, \frac{x}{Q^2}\, i\epsilon^{\mu\nu qP} \tag{10}$$

This projector picks up at tree level (Figure 1a) and after integrating over the phase space for one particle, only contributions proportional to delta functions in the convolution variables, being the proportionality factor

$$c_j = 4\pi Q^2_{q_j}\, \frac{\alpha^2}{S_H x} \tag{11}$$

Then,

$$\frac{d\Delta\sigma}{dx\,dy\,dz} = \lambda Y_P \sum_{j=q,\bar{q}} c_j \left\{ M_j(x,(1-x)z)\,(1-x) + \Delta f_j(x)\, D_j(z) \right\} \tag{12}$$

where we have dropped the indeces labelling the target and produced hadron. The virtual corrections (Figures 1b, 1c, 1d) give the same contribution but now multiplied by the usual factor [16]

$$\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} C_f \left(-2\frac{1}{\epsilon^2} - 3\frac{1}{\epsilon} - 8 - \frac{\pi^2}{3}\right) \tag{13}$$

The results for the real gluon emission (Figure 2) and the box diagrams (Figure 3) were calculated using the program TRACER [17] and can be found in apendix A. Notice that, as we are working in the HVBM scheme, terms proportional to the square of the $d-4$ dimensional component of the momentum of the outgoing particles, $\hat{p}^2_{\rm out}$, must be isolated [13]. Working in the photon-parton center of mass frame, there is no need to discriminate between the two outgoing particles, because they have opposite momenta. Furthermore, in this frame the incoming particles do not have $d-4$ components of the momentum. For fragmentation like configurations, the two-particle phase space in which the matrix elements are integrated, is given by

$$dPS^{(2)} = \frac{1}{8\pi}\frac{(4\pi)^{\epsilon}}{\Gamma(-\epsilon)}\frac{u-x}{u(1-x)}\,d\rho \int_0^{\hat{p}^2_{\rm max}} d\hat{p}^2_{\rm out}\left(\hat{p}^2_{\rm out}\right)^{-1-\epsilon} \tag{14}$$

with

$$\hat{p}^2_{\rm max} = Q^2(1-u)\,u\,(1-\rho)\left(\rho - \frac{x(1-u)}{u(1-x)}\right)\left(\frac{1-x}{u-x}\right)^2, \tag{15}$$

where the variable ρ is defined by

$$\rho \equiv \frac{E_\alpha}{E_N(1-x)} \tag{16}$$

i.e., the energy fraction of the parton α which undergoes hadronization. Due to the fact that all the contributions can be decomposed as

$$P^{\mu\nu}_{pol}\Delta H_{\mu\nu} = A(u,\rho) + B(u,\rho)\,\hat{p}^2_{\rm out}\,, \tag{17}$$

the phase space integration can be splitted into one part that is identical to the unpolarized case and another one which is purely $d-4$ dimensional. The results coming from the latter are singled out, writing them under hats, as they contain soft contributions which have to be factorized.

$$\begin{aligned} P^{\mu\nu}_{pol}\Delta H_{\mu\nu} dPS^{(2)} = \frac{1}{8\pi}\frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\,\frac{d\rho}{(1-a(u))}\left(\frac{Q^2(1-u)}{u}\right)^{-\epsilon}\left(\frac{(1-\rho)\,(\rho-a(u))}{(1-a(u))^2}\right)^{-\epsilon} \\ \left[A(u,\rho) - \frac{\epsilon}{1-\epsilon}\frac{(1-\rho)\,(\rho-a(u))}{(1-a(u))^2}\,\frac{Q^2(1-u)}{u}\,B(u,\rho)\right] \end{aligned} \tag{18}$$

As it has been shown in reference [9], the distinctive value

$$\rho = a(u) \equiv \frac{x(1-u)}{u(1-x)} \tag{19}$$

represents the configuration where the hadrons are produced in the direction of the incoming nucleon, thus giving rise to additional collinear singularities which do not show up neither in totally inclusive deep inelastic scattering nor in electron-proton annihilation. For fracture like configurations it is convenient to use the variable

$$\omega \equiv \frac{1-\rho}{1-a(u)} \tag{20}$$

which transforms equation (14) into the usual two particle phase space for HVBM [13].

Adding up contributions, we finally find

$$\begin{aligned}
\frac{d\Delta\sigma}{dx\,dy\,dz} &= Y^p \lambda \sum_{i=q,\bar{q}} c_i \left\{ \int\int_A \frac{du}{u}\frac{d\rho}{\rho} \left\{ \Delta q_i(\frac{x}{u})\, D_{q_i}(\frac{z}{\rho})\, \delta(1-u)\delta(1-\rho) \right.\right.\\
&+ \Delta q_i(\frac{x}{u})\, D_{q_i}(\frac{z}{\rho})\, \frac{\alpha_s}{2\pi} \left[-\frac{1}{\hat{\epsilon}} \left(P_{q\leftarrow q}(\rho)\delta(1-u) + \Delta P_{q\leftarrow q}(u)\delta(1-\rho) \right) + C_f \Delta\Phi_{qq}(u,\rho) \right]\\
&+ \Delta q_i(\frac{x}{u})\, D_g(\frac{z}{\rho})\, \frac{\alpha_s}{2\pi} \left[-\frac{1}{\hat{\epsilon}} \Big(P_{g\leftarrow q}(\rho)\delta(1-u) + \Delta\hat{P}_{gq\leftarrow q}(u)\delta(\rho-a) \Big) + C_f \Delta\Phi^A_{qg}(u,\rho) \right]\\
&+ \left.\Delta g(\frac{x}{u})\, D_{q_i}(\frac{z}{\rho})\, \frac{\alpha_s}{2\pi} \left[-\frac{1}{\hat{\epsilon}} \Big(\Delta P_{q\leftarrow g}(u)\delta(1-\rho) + \Delta\hat{P}_{q\bar{q}\leftarrow g}(u)\delta(\rho-a) \Big) + T_f \Delta\Phi_{gq}(u,\rho) \right] \right\}\\
&+ \int_B \frac{du}{u}(1-x) \left\{ \Delta M_{q_i}(\frac{x}{u},(1-x)z) \left(\delta(1-u) \;+ \frac{\alpha_s}{2\pi} \left[-\frac{1}{\hat{\epsilon}} \Delta P_{q\leftarrow q}(u) + C_f \Delta\Phi_q(u,\rho) \right] \right) \right.\\
&+ \left.\left. \Delta M_g(\frac{x}{u},(1-x)z)\, \frac{\alpha_s}{2\pi} \left[-\frac{1}{\hat{\epsilon}} \Delta P_{q\leftarrow g}(u) + T_f \Delta\Phi_g(u,\rho) \right] \right\} \right\}
\end{aligned} \tag{21}$$

where

$$\frac{1}{\hat{\epsilon}} \equiv \frac{1}{\epsilon} \frac{\Gamma[1-\epsilon]}{\Gamma[1-2\epsilon]} \left(\frac{4\pi\mu^2}{Q^2} \right)^{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \log(4\pi) + \log(\frac{\mu^2}{Q^2}) + \mathcal{O}(\epsilon) \tag{22}$$

The integration ranges for both convolutions, labelled A and B, come from the definition of the variables and momentum conservation and can be found in appendix B. The poles proportional to $\delta(1-u)$ correspond to final state singularities, so are multiplied by unpolarized Altarelli Parisi kernels $P_{i\leftarrow j}(\rho)$ [18]. Those proportional to $\delta(1-\rho)$, are related to the initial state singularities and are multiplied by spin dependent kernels $\Delta P_{i\leftarrow j}(u)$ [18]. The poles proportional to $\delta(\rho-a)$ are the collinear divergences mentioned previously and are multiplied by unsubstracted polarized splitting functions $\Delta\hat{P}_{ij\leftarrow k}(u)$ [19].

The functions $\Delta\Phi(u,\rho)$ are the finite next to leading order contributions to the cross section. $\Delta\Phi_{i\,j}(u,\rho)$ is a i (i = quark, gluon) initiated contribution where an outgoing parton j undergoes hadronization. $\Delta\Phi_{q\,(g)}(u,\rho)$ are the quark (gluon) initiated corrections to the fracture processes, which are identical to those of the totally inclusive polarized structure function. Notice that $\Delta\Phi_{q\,g}(u,\rho)$ depends explicitely on the integration subinterval and the others contain $()_+$ prescriptions and δ functions which have support in certain subintervals only. The expressions for the kernels and the finite NLO terms can be found in appendix C.

Factorization.

Having computed the whole cross section up to next to leading order, we are now able to factorize all the divergences and finite soft terms by means of the definition of scale dependent distributions. In order to respect the universal character of these distributions, it is mandatory to use here the same factorization prescriptions which were fixed in totally inclusive polarized deep inelastic scattering, for polarized parton distributions, and in one-particle inclusive electron-positron annihilation, for fragmentation functions. Provided this consistency requirement is satisfied, one can adopt any well defined prescription.

Fixing the factorization scale equal to Q^2, in the $\overline{MS}$ scheme the prescription ammounts to absorbe only the $1/\hat{\epsilon}$-terms. In the case of polarized deep inelastic scattering, it has been shown that this scheme leaves some soft contributions unsubstracted [13]. Within the HVBM prescription for γ_5 and $\epsilon_{\mu\nu\rho\sigma}$, these contributions can be identified because they come from the use of helicity projectors for the initial state partons [13]. They are related with the terms coming from de $d-4$ dimensional phase space integration (hat terms). It is possible then to define a slight variation of the traditional $\overline{MS}$ scheme, called $\overline{MS}_p$ [13, 14], in order to substract the remanent soft contributions. In general, the definition of the scale dependent quark distributions can be written as

$$\begin{aligned}\Delta q_i(\xi) &= \int_\xi^1 \frac{du}{u}\left\{\left[\delta(1-u)+\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}\Delta P_{q\leftarrow q}(u)-C_f\Delta\tilde{f}_q^F(u)\right)\right]\Delta q_i(\frac{\xi}{u},Q^2)\right.\\ &\quad \left.+\frac{\alpha_s}{2\pi}\left[\frac{1}{\hat{\epsilon}}\Delta P_{q\leftarrow g}(u)-T_f\Delta\tilde{f}_g^F(u)\right]\Delta g(\frac{\xi}{u},Q^2)\right\} \qquad (23)\end{aligned}$$

In the $\overline{MS}_p$, the finite substraction term $\Delta\tilde{f}_q^F$, is designed to absorb soft contributions coming from real gluon emission diagrams and enforces the non-singlet axial current conservation. Conversely, the term $\Delta\tilde{f}_g^F$, which absorbes soft contributions coming from photon-gluon fusion diagrams, leads to the axial anomaly result for the singlet axial current [20]. In this way, the $\overline{MS}_p$ definition of polarized parton distributions guarantees the conservation of $\Delta\Sigma=\sum_i\int_0^1\Delta q_i(x,Q^2)dx$, which implies the scale independence of the net spin carried by quarks.

As there is no need to make finite substractions for *unpolarized* final states, the definition for the scale dependent fragmentation functions is simply given by

$$D_{q_i}(\xi) = \int_\xi^1 \frac{du}{u}\left\{\left[\delta(1-u)+\frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}}P_{q\leftarrow q}(u)\right]D_{q_i}(\frac{\xi}{u},Q^2)+\frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}}P_{g\leftarrow q}(u)\,D_g(\frac{\xi}{u},Q^2)\right\} \qquad (24)$$

which is the canonical $\overline{MS}$ prescription used in $\mathrm{e}^+\,\mathrm{e}^-\to\mathrm{h}\,\mathrm{X}$.

For polarized fracture functions, the definition of the scale dependent distributions requires two parts, as in the unpolarized case. One, called homogeneus, which deals with initial state singularities in fracture like events, and another one called inhomogeneus, which has to absorb the additional collinear

singularities related to the $\rho = a(u)$ fragmentation configurations. In order to be able to substract the finite soft contributions that arise along the initial state divergences in the HVBM prescription, we also include the $\overline{MS}_p$ counterterms $\Delta\tilde{f}^{MH}$ and $\Delta\tilde{f}^{MI}$, so

$$\begin{aligned}
\Delta M_{q_i}(\xi,\zeta) = \int_{\frac{\xi}{1-\zeta}}^{1} \frac{du}{u} \Bigg\{ & \left[\delta(1-u) + \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}\Delta P_{q\leftarrow q}(u) - C_f \Delta\tilde{f}_q^{MH}(u)\right)\right] \Delta M_{q_i}(\frac{\xi}{u},\zeta,Q^2) \\
& + \frac{\alpha_s}{2\pi}\left[\frac{1}{\hat{\epsilon}}\Delta P_{q\leftarrow g}(u) - T_f \Delta\tilde{f}_g^{MH}(u)\right] \Delta M_g(\frac{\xi}{u},\zeta,Q^2)\Bigg\} \\
+ \int_{\xi}^{\frac{\xi}{\xi+\zeta}} \frac{du}{u}\frac{u}{x(1-u)} \Bigg\{ & \frac{\alpha_s}{2\pi}\left[\frac{1}{\hat{\epsilon}}\Delta\hat{P}_{gq\leftarrow q}(u) - C_f \Delta\tilde{f}_q^{MI}(u)\right] \Delta q_i(\frac{\xi}{u},Q^2)\, D_g(\frac{\zeta u}{\xi(1-u)},Q^2) \\
& + \frac{\alpha_s}{2\pi}\left[\frac{1}{\hat{\epsilon}}\Delta\hat{P}_{q\bar{q}\leftarrow g}(u) - \frac{\alpha_s}{2\pi} T_f \Delta\tilde{f}_g^{MI}(u)\right] \Delta g(\frac{\xi}{u},Q^2)\, D_{q_i}(\frac{\zeta u}{\xi(1-u)},Q^2)\Bigg\}
\end{aligned} \tag{25}$$

For the homogeneus part, the counterterms $\Delta\tilde{f}^{MH}(u)$ are the same as those used in polarized inclusive DIS, because the structure of the corrections is identical. For the inhomogeneus part, the counterterms are those associated with the ones found previously for the homogeneus part, as can be seen in the hat terms of the finite contributions. This is so because the finite contributions and divergences come from the same real gluon emission and quark-antiquark gluon splitting, The substracted cross section can then be written as

$$\begin{aligned}
\frac{d\Delta\sigma}{dx\,dy\,dz} &= Y^p\lambda \sum_{i=q,\bar{q}} c_i \Bigg\{ \int\int_A \frac{du}{u}\frac{d\rho}{\rho} \Bigg\{ \Delta q_i(\frac{x}{u},Q^2)\, D_{q_i}(\frac{z}{\rho},Q^2)\,\delta(1-u)\delta(1-\rho) \\
&+ \Delta q_i(\frac{x}{u},Q^2)\, D_{q_i}(\frac{z}{\rho},Q^2)\,\frac{\alpha_s}{2\pi} C_f \left[\Delta\Phi_{qq}(u,\rho) - \Delta\tilde{f}_q^F(u,\rho)\right] \\
&+ \Delta q_i(\frac{x}{u},Q^2)\, D_g(\frac{z}{\rho},Q^2)\,\frac{\alpha_s}{2\pi} C_f \left[\Delta\Phi_{qg}^A(u,\rho) - \Delta\tilde{f}_q^{MI}(u,\rho)\right] \\
&+ \Delta g(\frac{x}{u},Q^2)\, D_{q_i}(\frac{z}{\rho},Q^2)\,\frac{\alpha_s}{2\pi} T_f \left[\Delta\Phi_{gq}(u,\rho) - \Delta\tilde{f}_g^F(u,\rho) - \Delta\tilde{f}_g^{MI}(u,\rho)\right]\Bigg\} \\
&+ \int_B \frac{du}{u}(1-x)\Bigg\{\Delta M_{q_i}(\frac{x}{u},(1-x)z,Q^2)\left(\delta(1-u) + \frac{\alpha_s}{2\pi} C_f \left[\Delta\Phi_q(u,\rho) - \Delta\tilde{f}_q^{MH}(u)\right]\right) \\
&+ \Delta M_g(\frac{x}{u},(1-x)z,Q^2)\,\frac{\alpha_s}{2\pi} T_f \left[\Delta\Phi_g(u,\rho) - \Delta\tilde{f}_g^{MH}(u)\right]\Bigg\}\Bigg\}
\end{aligned} \tag{26}$$

where the counterterms in the $\overline{MS}_p$ scheme are given by

$$\begin{aligned}
\Delta\tilde{f}_q^F(u,\rho) &= 4(u-1)\,\delta(1-\rho) \\
\Delta\tilde{f}_q^{MI}(u,\rho) &= 4(u-1)\,\delta(\rho-a) \\
\Delta\tilde{f}_q^{MH}(u) &= 4(u-1) \\
\Delta\tilde{f}_g^F(u,\rho) &= 2(1-u)\,\delta(1-\rho) \\
\Delta\tilde{f}_g^{MI}(u,\rho) &= 2(1-u)\,\delta(\rho-a) \\
\Delta\tilde{f}_g^{MH}(u) &= 2(1-u)
\end{aligned} \tag{27}$$

in the case of the light quarks (u, d, s) and 0 for heavy quarks [14]. Notice that in the $\overline{MS}$ scheme, all

of these counterterms are choosen to be 0.

Conclusions.

We have calculated the $\mathcal{O}(\alpha_s)$ one-particle inclusive cross section in polarized deep inelastic lepton-hadron scattering, showing that with the inclusion of polarized fracture functions it is possible to consistently factorize all the collinear singularities that occur and that, within the HVBM prescription, the $\overline{MS}_p$ scheme can be straightforwardly applied in order to factorize unwanted finite soft contributions. In this way, the $\overline{MS}_p$ scheme guarantees the conservation of the non-singlet axial current and the non-conservation of the singlet one, as dictated by the anomaly result. This requirement allows the definition of polarized parton and fracture distributions intimately related to the fraction of the nucleon spin carried by partons.

Having defined an universal and physically meaningfull factorization scheme for both current and target fragmentation, consistent with those used in totally inclusive spin dependent deep inelastic scattering and unpolarized electron proton annihiliation, it will be possible to perform an unanmbiguous $\mathcal{O}(\alpha_s)$ analysis of forthcoming inclusive experiments.

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Appendix A.

The projection of the real gluon emission matrix element, within the HVBM prescription, is given by

$$
\begin{aligned}
P^{\mu\nu}_{pol}\Delta H_{\mu\nu} &= -4\pi\frac{\alpha_s}{2\pi}Q_q^2 2\pi\frac{\alpha^2}{S_H x}C_f\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{u}{Q^2} \\
&\times \left[\frac{4\,(1+\epsilon)\,(s_{ig}-s_{iq})\,s_{qg}}{s_{ig}}+\frac{4\,(-1+\epsilon)\,s_{ig}\,(s_{ig}+s_{iq})}{s_{qg}}-8\,\epsilon\,s_{ig}\right. \\
&- \left.\frac{8\,s_{iq}\,\left({s_{ig}}^2+Q^2\,s_{iq}+s_{ig}\,s_{iq}\right)}{s_{ig}\,s_{qg}}-\frac{8\,(s_{ig}+s_{iq})^2\,(-s_{ig}+\epsilon\,s_{ig}-s_{qg}-\epsilon\,s_{qg})\,\hat{p}^2_{out}}{{s_{ig}}^2\,s_{qg}}\right]
\end{aligned}
\tag{28}
$$

where $s_{AB} = 2\,p_A{\cdot}p_B$. The labels q and g take the values 1 and 2, respectively, for quark fragmentation, or 2 and 1 for gluon fragmentation.

For photon gluon fusion

$$
\begin{aligned}
P^{\mu\nu}_{pol}\Delta H_{\mu\nu} &= 4\pi\frac{\alpha_s}{2\pi}Q_q^2 2\pi\frac{\alpha^2}{S_H x}T_f\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \\
&\times \left[\frac{-4\,\left(-2\,Q^2+s_{iq}+s_{i\bar{q}}\right)\,\left({s_{iq}}^2+{s_{i\bar{q}}}^2\right)\,u}{Q^2\,s_{iq}\,s_{i\bar{q}}}+\frac{8\,(s_{iq}+s_{i\bar{q}})^2\,\left({s_{iq}}^2+{s_{i\bar{q}}}^2\right)\,u\,\hat{p}^2_{out}}{Q^2\,{s_{iq}}^2\,{s_{i\bar{q}}}^2}\right]
\end{aligned}
\tag{29}
$$

The expression for s_{AB} in terms of the variables ρ, u and ω can be found in reference [9].

Appendix B.

The integration range A is splitted into two subintervals

$$
A_1\,:\,u\in\left[x\,,\frac{x}{x+(1-x)z}\right]\,,\quad \rho\in[a(u)\,,1]
\tag{30}
$$

and

$$
A_2\,:\,u\in\left[\frac{x}{x+(1-x)z}\,,1\right]\,,\quad \rho\in[z\,,1]
\tag{31}
$$

while B is given by

$$
B\,:\,u\in\left[\frac{x}{x-(1-x)z}\,,1\right]
\tag{32}
$$

Appendix C.

The splitting functions are given by [18, 19]

$$
\begin{aligned}
\Delta P_{q\leftarrow q}(u) &= C_f\left[2\left(\frac{1}{1-u}\right)_+ + \frac{3}{2}\delta(1-u)-1-u\right]\\
\Delta P_{q\leftarrow g}(u) &= T_f\left[2u-1\right]\\
\Delta \hat{P}_{gq\leftarrow q}(u) &= C_f\left[\frac{1+u^2}{1-u}\right]\\
P_{q\leftarrow q}(u) &= C_f\left[2\left(\frac{1}{1-u}\right)_+ + \frac{3}{2}\delta(1-u)-1-u\right]\\
P_{g\leftarrow q}(u) &= C_f\left[2\frac{1}{u}-2+u\right]\\
\Delta \hat{P}_{q\bar{q}\leftarrow g}(u) &= T_f\left[2u-1\right]
\end{aligned}
\tag{33}
$$

The finite next to leading order contributions are

$$
\begin{aligned}
&\Delta\Phi_{qq}(u,\rho) =\\
-\;& 8\,\delta(1-r)\delta(1-u) + \left[(1-r)+\frac{1+r^2}{1-r}\log(r)-(1+r)\log(1-r)+2\left(\frac{\log(1-r)}{(1-r)}\right)_+\right]\delta(1-u)\\
+\;& \left[-(1-u)+\frac{1+u^2}{1-u}\log(\frac{1-x}{u-x})-(1+u)\log(1-u)+2\left(\frac{\log(1-u)}{(1-u)}\right)_+ - 2\widehat{(1-u)}\right]\delta(1-r)\\
+\;& 2\left(\frac{1}{1-r}\right)_+\left(\frac{1}{1-u}\right)_+ - \left(\frac{1}{1-r}\right)_+(1+u)-\left(\frac{1}{1-u}\right)_+(1+r)-\frac{2\;(1-u)\;u\;(1-x)}{u-x}\\
-\;& \frac{(1-r)\;(1-u+(1-x)\;(-1+u\;(1+2\,u)\;(1-x)-x))}{(u-x)^2}+\frac{4\,u\;(1-x)}{u-x}-\frac{2\,x}{u-x}
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
\Delta\Phi^{A=1}_{qg}(u,\rho) &= \delta(\rho-a)\left[-2\widehat{(1-u)}-(1-u)+\frac{1+u^2}{1-u}\log\left(\frac{(1-x)(1-u)}{(u-x)}\right)\right]\\
&\quad+\left(\frac{1}{\rho-a}\right)_+\frac{1+u^2}{1-u}-\frac{\rho\,u^2\,(1-x)^2}{(u-x)^2}+\frac{\rho\,u^3\,(1-x)^2}{(1-u)\;(u-x)^2}\\
&\quad-\frac{2\,u^3\;(1-x)}{(1-u)\;(u-x)}+\frac{u\;(1-x)\;x}{(u-x)^2}-\frac{2\,u^2\;(1-x)\;x}{(u-x)^2}
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
\Delta\Phi^{A=2}_{qg}(u,\rho) &= \delta(1-u)\left[\rho+\left(\rho+\frac{2}{\rho}-2\right)\log\left(\rho(1-\rho)\right)\right]+\left(\frac{1}{1-u}\right)_+\left(\rho+\frac{2}{\rho}-2\right)\\
&\quad-\frac{2\,(1-\rho)^2}{\rho\;(1-u)}+\frac{1-u}{r-a}-\frac{2\;(1-u)\;u\;(1-x)}{u-x}+\frac{2\,(1-\rho)^2\,u^3\,(1-x)^2}{(\rho-a)\;(1-u)\;(u-x)^2}\\
&\quad+\frac{(1-\rho)\;u\;(1-x)\;x}{(u-x)^2}+\frac{(2-\rho)\;x}{u-x}
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
\Delta\Phi_{gq}(u,\rho) &= \left(\delta(1-\rho)+\delta(\rho-a)\right)\left[2\widehat{(1-u)}+(2u-1)\log\left(\frac{(1-x)(1-u)}{(u-x)}\right)\right]\\
&\quad+(2u-1)\left[\left(\frac{1}{1-\rho}\right)_+ + \left(\frac{1}{\rho-a}\right)_+ - 2u\frac{1-x}{u-x}\right]
\end{aligned}
\tag{37}
$$

$$\Delta\Phi_q(u,\rho) = -\frac{1+u^2}{1-u}\log(u) - (1+u)\log(1-u) + 2\left(\frac{\log(1-u)}{(1-u)}\right)_+ - \frac{3}{2}\left(\frac{1}{1-u}\right)_+ + 3u + \frac{7}{2}\delta(1-u) - 2(\widehat{1-u}) \tag{38}$$

$$\Delta\Phi_g(u,\rho) = 2(\widehat{1-u}) + (2u-1)\log\left(\frac{(1-u)}{u} - 1\right) \tag{39}$$

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Figure Captions

Figure 1   a) Lowest order parton-photon graph ; b),c) and d) virtual gluon correction graphs to a).

Figure 2   Real gluon emission corrections to 1a).

Figure 3   Gluon contribution at order α_s

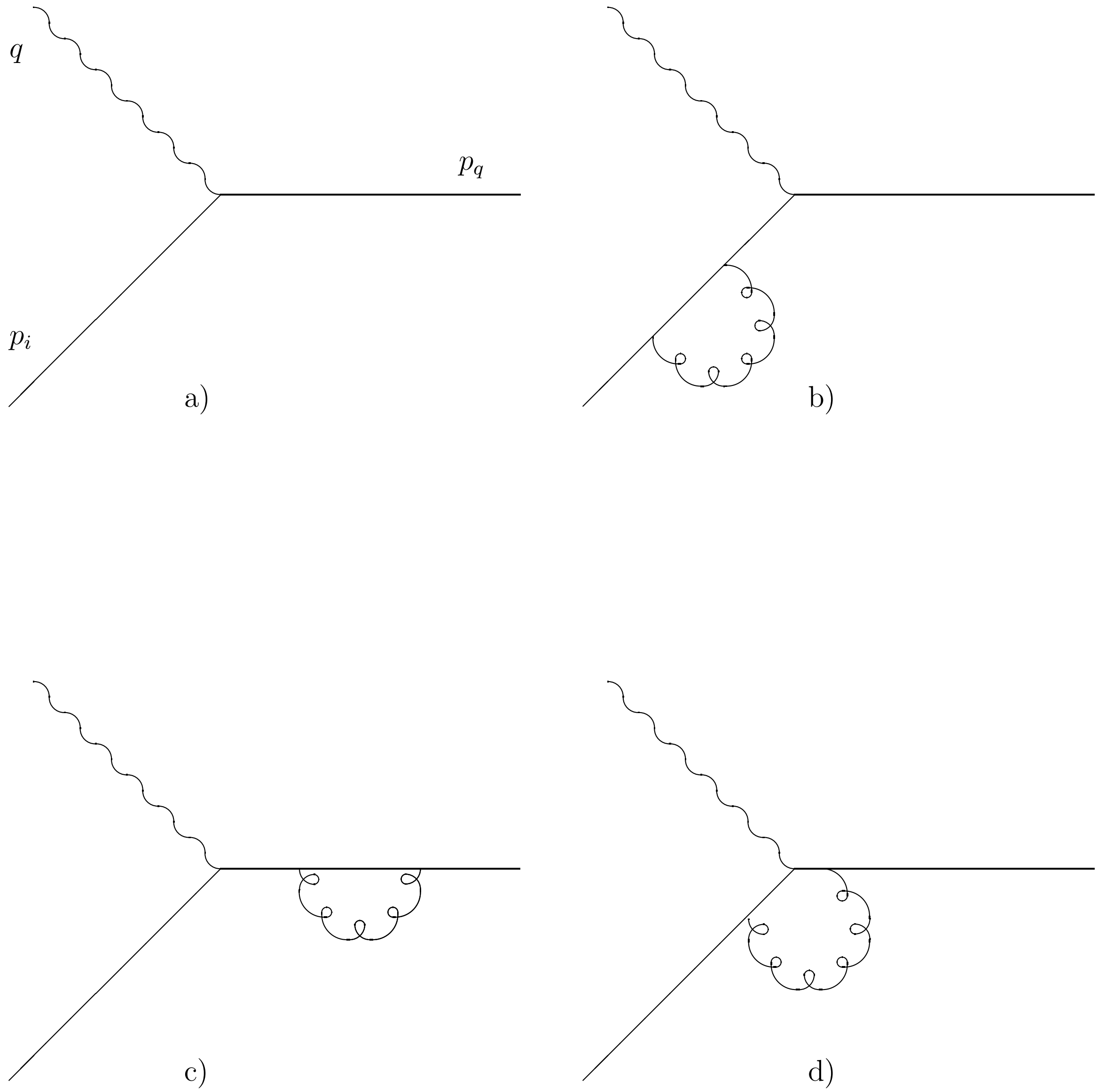

Figure 1

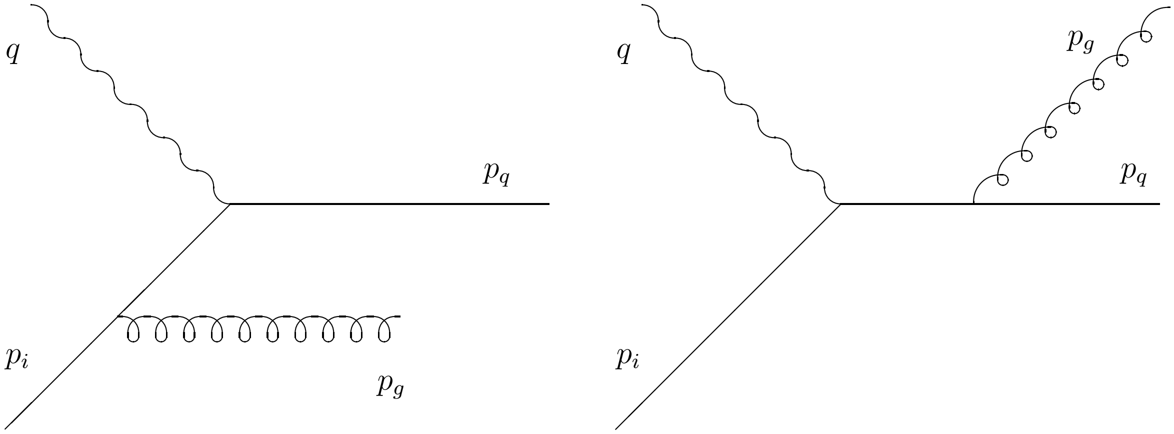

Figure 2

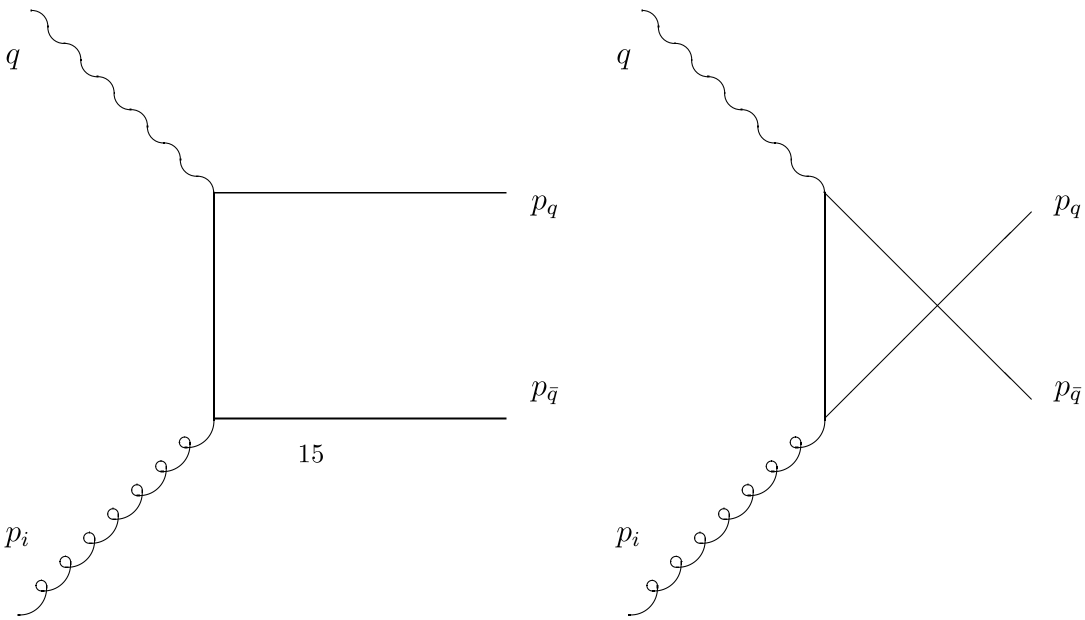

Figure 3